

Segmenting Planar Superpixel Adjacency Graphs w.r.t. Non-planar Superpixel Affinity Graphs – Supplement –

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1 Derivation of the Dual of the Decomposition

We now derive the dual of the Lagrangian decomposition. We begin by writing the decomposition below.

$$\max_{\theta^{pmc}, \psi} 1^T(\theta^P - \theta^{pmc} - Y\psi) + 1^T(\psi + \theta^{NP}) \quad (1)$$

$$\text{subject to } Z\theta^{pmc} \geq 0 \quad (2)$$

$$-W\psi \geq \theta^{NP} \quad (3)$$

$$\theta^P - \theta^{pmc} - Y\psi \leq 0 \quad (4)$$

$$\min([0, -\theta^P]) \leq -\theta^{pmc} - Y\psi \quad (5)$$

$$\psi \geq 0 \quad (6)$$

The dual of this decomposition for the planar multicut problem is derived in [1]. In order to be closer to the form of [1], we make the following adjustments to the notation. We define $\phi = \min([0, -\theta^P])$ and $\hat{\lambda} = -\theta^{pmc} - \phi$. Notice that $\theta^{pmc} = -\hat{\lambda} - \phi$. We now rewrite the decomposition using $\hat{\lambda}$ and ϕ .

$$\max_{\hat{\lambda}, \psi} 1^T(\theta^P + \hat{\lambda} + \phi - Y\psi) + 1^T(\psi + \theta^{NP}) \quad (7)$$

$$\text{subject to } Z(-\hat{\lambda} - \phi) \geq 0 \quad (8)$$

$$-W\psi \geq \theta^{NP} \quad (9)$$

$$\theta^P + \hat{\lambda} + \phi - Y\psi \leq 0 \quad (10)$$

$$\phi \leq \hat{\lambda} + \phi - Y\psi \quad (11)$$

$$\psi \geq 0 \quad (12)$$

We now move terms to different sides of the relevant inequalities. This facilitates the writing of the LP as a Lagrangian.

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$$\max_{\hat{\lambda}, \psi} 1^T(\theta^P + \hat{\lambda} + \phi - Y\psi) + 1^T(\psi + \theta^{\text{NP}}) \quad (13)$$

$$\text{subject to } 0 \leq -Z\hat{\lambda} - Z\phi \quad (14)$$

$$0 \leq -\theta^{\text{NP}} - W\psi \quad (15)$$

$$0 \leq -\theta^P - \phi + Y\psi - \hat{\lambda} \quad (16)$$

$$0 \leq \hat{\lambda} - Y\psi \quad (17)$$

$$\psi \geq 0 \text{ .} \quad (18)$$

We now write the above LP as a Lagrangian.

$$\min_{\gamma, \omega, \beta, \delta \geq 0} \max_{\hat{\lambda}, \psi \geq 0} 1^T(\theta^P + \hat{\lambda} + \phi - Y\psi) + 1^T(\psi + \theta^{\text{NP}}) \quad (19)$$

$$+ \gamma^T(-Z\hat{\lambda} - Z\phi) \quad (20)$$

$$+ \omega^T(-\theta^{\text{NP}} - W\psi) \quad (21)$$

$$+ \beta^T(-\theta^P - \phi + Y\psi - \hat{\lambda}) \quad (22)$$

$$+ \delta^T(\hat{\lambda} - Y\psi) \text{ .} \quad (23)$$

Now we group the terms that have components of ψ and $\hat{\lambda}$ together. This facilitates the creation of new constraints.

$$\min_{\gamma, \omega, \beta, \delta \geq 0} 1^T(\theta^P + \phi) + 1^T\theta^{\text{NP}} - \gamma^T Z\phi - \omega^T\theta^{\text{NP}} + \beta^T(-\theta^P - \phi^T) \quad (24)$$

$$+ (1^T - \gamma^T Z - \beta^T + \delta^T)\hat{\lambda} \quad (25)$$

$$+ (-1^T Y + 1^T - \omega^T W + \beta^T Y - \delta^T Y)\psi \text{ .} \quad (26)$$

Since $\hat{\lambda}$ is unbounded and ψ is non-negative, the LP with the following constraints is equivalent to the Lagrangian above.

$$\min_{\gamma, \omega, \beta, \delta \geq 0} 1^T(\theta^P + \phi) + 1^T\theta^{\text{NP}} - \gamma^T Z\phi - \omega^T\theta^{\text{NP}} + \beta^T(-\theta^P - \phi^T) \quad (27)$$

$$\text{subject to } \gamma^T Z + \beta^T = 1^T + \delta^T \quad (28)$$

$$\omega^T W \geq \beta^T Y - 1^T Y + 1^T - \delta^T Y \text{ .} \quad (29)$$

Except for transposes of some terms, this is the final form of the Lagrangian decomposition used in the main manuscript.

References

1. J. Yarkony. *MAP inference in Planar Markov Random Fields with Applications to Computer Vision*. PhD thesis, University of California, Irvine, 2012.